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by

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SHOCK WAVES IN INTERPLANETARY PLASMA

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The present work is aimed toward understanding (1) the structure of the shock wave around the earth which has been postulated by Axford[1] and the author[2], and (2) the front of an approaching solar plasma cloud, with a hope of improving the predictions of Parker's shock[3] wave theory with regard to Forbush decreases. Under the conditions in which we are interested, collisions are completely negligible. One the other hand, work by Adlam and Allen[4] on the structure of strong pulses in plasma with no dissipation mechanism showed that no shocks are formed; instead the plasma returns to its initial state after passage of the pulse. It is necessary to have a dissipation mechanism, therefore, and in the present work it is provided by the two-stream plasma instability[5] which we now describe.

Consider an element of the solar wind plasma as it approaches the front of a shock wave. As this element encounters the magnetic field change due to the shock, the electrons and ions will be deflected in opposite directions; if the relative velocity is sufficiently large compared to the electron thermal velocity, the system will be unstable as follows: suppose that the ions are at rest, and that the electrons have velocity V. We neglect the thermal velocities of both particles. Suppose that the ion density is not quite uniform, but has a perturbation $\Delta n = \delta n \sin kx$. This density perturbation will make an electric field $E_o \cos kx$, where $E_o = -\frac{4\pi e}{k}\delta n$, and a potential $+\frac{4\pi e}{k^2}\delta n \sin kx$. If the perturbation is a very slowly growing one, then the electrons will approximately conserve energy, so their velocity will be $v = V + \frac{4\pi e^2}{mk^2V}\delta n \sin kx$. Again for slowly growing perturbations, the flux $n_e v$ of electrons will be approximately constant, so that the perturbation electron density will be:

$$\Delta n_e = -\frac{4\pi N_e e^2}{m k^2 V^2}\,\delta n \sin kx = -\frac{\omega_{pe}^2}{k^2 V^2}\,\delta n \sin kx$$

Thus, electrons pile up to give charge density of the same sign as that which started the process, and the feedback is positive. It is greater than one if k is sufficiently small, and so the system is unstable.

Calculations of the growth rate, taking into account the thermal velocities of the particles and without our simplifying assumption of slow growth rate, have been made by several authors. In Figure 1. we present the calculation of the growth rate for zero temperature as given by Buneman[5]. It can be shown that the growth rate of the fastest growing waves is given by:

$$IP(\omega_{MAX}) \equiv \Omega = .66\,\frac{\omega_{pe}}{\left(\frac{M}{m}\right)^{\frac{1}{3}}} \tag{1}$$

in the limit of large ion-electron mass ratio $\frac{M}{m}$. To take thermal motion into account, we have used the following simple function fitted to the results of Stringer[6] for the growth rate of the fastest growing waves in hydrogen:

$$IP(\omega_{MAX}) = \omega_{pi}\left[.22\left(\frac{V}{v_{te}}\right)^2 - .25\left(\frac{T_i}{T_e}\right)^2\right] \quad \text{if this is between 0 and } \Omega \tag{2}$$

$$= 0 \text{ or } \Omega \text{ otherwise.}$$

Here ω_{pe} and ω_{pi} are the electron and ion plasma frequencies, v_{te} the electron thermal velocity, and T_e and T_i the electron and ion temperatures.

An important characteristic of this formula is that if the electrons are hot enough:

$$\frac{v_{te}}{V} > \sqrt{\frac{.22}{.25}\,\frac{T_e}{T_i}}$$

the system is stable and the electric fields do not develop.

Waves will grow at an appreciable rate for a range of k values. Therefore, the plasma instability generates a system of fluctuating electric fields, which will act randomly on the electrons and ions, increasing their thermal energy until they become hot enough that the system is stable again. This thermal energy must be taken out of the ordered, streaming energy, and therefore, must act as a kind of friction on the streaming motion.

The growth rate of these oscillations is of the order of ω_{pi}. On the other hand, from the work of Adlam and Allen, the thickness of the shock front is of order $\frac{c}{\omega_{pe}}$ and so an element of the plasma requires a time $\frac{c}{\omega_{pe} v}$, where v is the velocity of the shock, to traverse the pulse. In the limit that $\frac{v}{c}$ is very small the counter streaming oscillations grow through many powers of e during the time that an element of plasma traverses the pulse and therefore it seems reasonable to replace the effect of the instability by a smooth friction which is the average of the effect of the instability over many growth periods.

In order to verify that these fluctuation fields do convert streaming motion into thermal motion, the Boltzmann equation for a system of electrons streaming through protons has been integrated on a large electronic computer. In these calculations, the magnetic field can be neglected if the Alfven velocity v_A is very small compared to $\sqrt{\frac{m}{M}}\,c$ where m and M are the electron and ion masses. The results show also, and this is an important distinguishing characteristic of this shock theory, that much more thermal energy is given to the electrons than to the ions, in the ratio $\sqrt{\frac{M}{m}}$ [7].

We, therefore, set up the equations of motion for a system of electrons and ions, acted upon by electromagnetic forces, by pressure gradients, and by friction parallel to the relative electron-ion velocity.

$$v_x \frac{d\vec{v}_e}{dx} = -\frac{e}{m}\left[\vec{E} + \frac{\vec{v}_e}{c}\times\vec{B}\right] - \frac{1}{m n_e}\nabla p_e - \frac{1}{m}(\vec{v}_e - \vec{v}_i)\, f \qquad (4a)$$

$$v_x \frac{d\vec{v}_i}{dx} = \frac{e}{M}\left[\vec{E} + \frac{\vec{v}_i}{c}\times B\right] - \frac{1}{M n_i}\nabla p_i - \frac{1}{M}(\vec{v}_i - \vec{v}_e)\, f$$

$$(\nabla\times B)_{y,z} = \frac{4\pi e}{c}\left[n_i \vec{v}_i - n_e \vec{v}_e\right]$$

$$\frac{d}{dx}(n_e v_{ex}) = \frac{d}{dx}(n_i v_{ix}) = \frac{dE_y}{dx} = \frac{dE_z}{dx} = 0$$

We have taken the x axis normal to the shock front.

We take the magnitude of the friction to be such as to decrease the relative streaming motion by a factor of e in N growth periods, [Eq (2)]

$$f = \frac{1}{N}\left(\frac{Mm}{M+m}\right) IP\left(\omega_{MAX}\right) \qquad (5)$$

The magnitude of the friction enters the theory only in the combination $\beta_A = N\frac{v_A}{c}$, so we can leave N to be determined experimentally, but expect it to be of order 1. The phenomenological friction takes energy out of the streaming motion and puts it into thermal motion, so to conserve energy we need to add terms to the pressure equations:

$$\frac{d}{dx}\left(p_e v_{ex}^{5/3}\right) = g_e \qquad (4b)$$

$$\frac{d}{dx}\left(p_i v_{ix}^{5/3}\right) = g_i$$

In order to conserve energy we must have the following relation between f and the g's.

$$g_e + g_i = \frac{2}{3} m\,(\vec{v}_e - \vec{v}_i)^2\, f \qquad (6)$$

The ratio between g_e and g_i is not determined by the theory which has been written down so far. It must be taken from the computer calculations on the streaming of electrons through ions mentioned above. It is found that:

$$\frac{g_e}{g_i} = \sqrt{\frac{M}{m}} \tag{7}$$

From this work also we get a relation between the friction and the mean square fluctuating electric field which is as follows:

$$\overline{E^2} \simeq \frac{4\pi m(\vec{v}_e - \vec{v}_i)^2}{\Omega} \tag{8}$$

where Ω is defined in equation (1).

We put these equations in dimensionless form by dividing all velocities by the Alfven velocity $B_0 / \sqrt{4\pi M_0 (M+m)}$, all magnetic fields by B_0, the magnitude of the field in front of the shock, the pressure by $\frac{6}{5} M_A \frac{B_0^2}{8\pi}$ and use $\frac{c}{\omega_p}$ as a unit of distance. M_A is the ratio of shock speed to Alfven speed. We also use the nonrelativistic limit which implies, as was shown by Adlam and Allen, that $n_i = n_e$ and $v_{ix} = v_{ex} \equiv v_x$.

We look for one-dimensional solutions of the resulting equations, on the assumption that the deviations from the equilibrium parameters fall off exponentially at large distances in front of the wave. This last requirement, as usual, imposes a restriction on the speed of the shock wave. In order to propagate perpendicular to magnetic field, the speed of the shock must be greater than the Alfven speed, but in order to propagate parallel to a magnetic field, the speed must be greater than $\sqrt{\frac{M}{m}}\, v_A$. The speed of the interplanetary solar wind is not this high, only about 7 v_A, so that this theory is capable of treating only a limited part of the earth's shock wave, where the angle between the interplanetary field and the shock front is sufficiently small. There is no reason to believe that the structure of the shock over the rest of the front is drastically different; however, the deviations of the shock quantities fall off as powers of r instead of exponentially in front of the shock.

Not only the critical shock speed, but other parameters of the shock depend strongly on the angle between the shock plane and the magnetic field. It is to be expected that this strong control of the shock by the magnetic field direction will influence the flow behind the shock. If the interplanetary magnetic field is not radial from the sun, therefore, there is no reason to expect the shock to be symmetrical around the earth-sun line, and therefore the magnetosphere also may be expected to be unsymmetrical.

There is another limitation on the present theory. The equation written above for the x component of velocity involves the derivatives of pressure gradient and the equation for pressure gradient involves the x component of velocity. When these equations are solved for their respective derivatives the denominator of the right hand side contains a factor $u_x - p$ which means that the right hand sides of the equations become infinite when the flow changes from supersonic to subsonic, that is, when an ordinary gas shock is formed. We have not included any viscosity in our equations therefore, we cannot treat the structure of this ordinary shock. The equations blow up and that's all there is to it. Clearly, however, the approximation which we have made so far, namely, that all of the heat generated by the friction is added to the gas right at the point where it is made, break down when the derivatives of the parameters become large just as viscosity is no longer negligible when the derivatives of the flow velocity become large in ordinary shock theory. We, therefore, need to make a more refined theory taking into account an effective viscosity and a heat flow. This has not yet been done. It turns out that we can still treat shocks that are sufficiently weak because no transition from supersonic to subsonic flow takes place. This is another restriction on our theory which again is considerable but not fatal when applied to the interplanetary space. The pressures which have been used in the figures 2. and 3. have been taken somewhat higher than those which are actually observed to prevent a transition to subsonic flow. Again, the correct equations would not be expected to be qualitatively different. There will be some additional heating both of electrons and ions due to the compression by the ordinary shock.

A typical solution for a shock propagating perpendicular to a magnetic field is shown as Figure 2. M_A, the ratio of shock speed to Alfven speed, is 1.95. A shock propagating parallel to the magnetic field, at $M_A = 22.4$ is shown in Figure 3. P_R is the ratio of electron pressure to ion pressure and P is the total pressure in units of $\frac{6}{5} M_A \frac{B_0^2}{8\pi}$. The quantity F is proportional to the friction and is defined by:

$$F = \frac{1}{m \omega_p \beta_A} f \tag{9}$$

In terms of F, the mean square electric field is:

$$E_0^2 = 4\pi m (v_e - v_i)^2 m \frac{\omega_p}{\Omega} \beta_A F \tag{10}$$

The ion transverse velocities are not shown. They are very small, of the order of $\frac{m}{M}$ times the electron transverse velocities.

Such shocks, then, will convert part of the incoming energy of the solar wind into thermal energy, principally of the electrons, behind the shock. Such an intense flux of electrons, between the magnetosphere boundary and the expected position of the shock front, has been found experimentally[8].

The shock also develops a system of waves behind it something like Alfven waves which are stationary with respect to the shock front. They are, in fact, waves of the type investigated by Hain, Lüst and Schlüter[9]. The transverse magnetic field in these waves is generally considerably larger than the original magnetic field in the unshocked gas. Such magnetic fields are observed. The measurements made by Sonett[10] and coworkers in Pioneers I and V, showed very strong magnetic pulses irregularly spaced in time, but with a period of the order of 10 seconds.

As we have seen, the pulses which we calculate are stationary with respect to the shock front. However, the shock, because of fluctuations in the interplanetary field, is presumably moving in and out with respect to the earth at a speed which is probably larger than the satellite speed and therefore will cause the pulses to pass over the satellite at a velocity which is unknown. Therefore, the separation of the pulses in time will be unknown. For typical solar wind parameters the distance between pulses, which would be calculated on this theory is of the order of 10 kilometers. Pioneers I and V measured pulses with periods of about 10 seconds which would correspond to a velocity of the satellite relative to the shock front of 1 kilometer per second. This seems of the right order of magnitude. Cahill [11] has also measured a magnetic field in the probable post-shock region which is several times larger than that in the unshocked solar wind.

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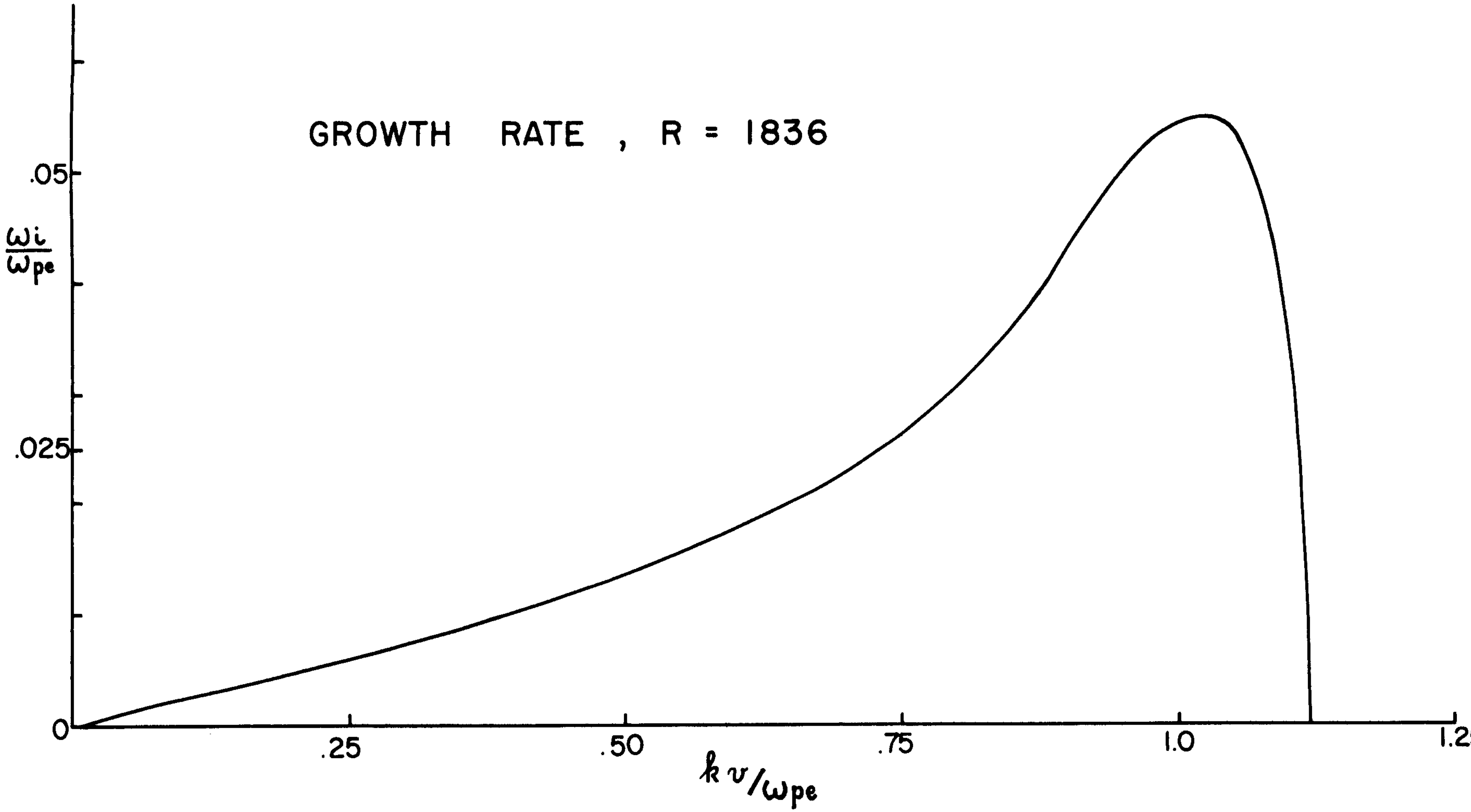
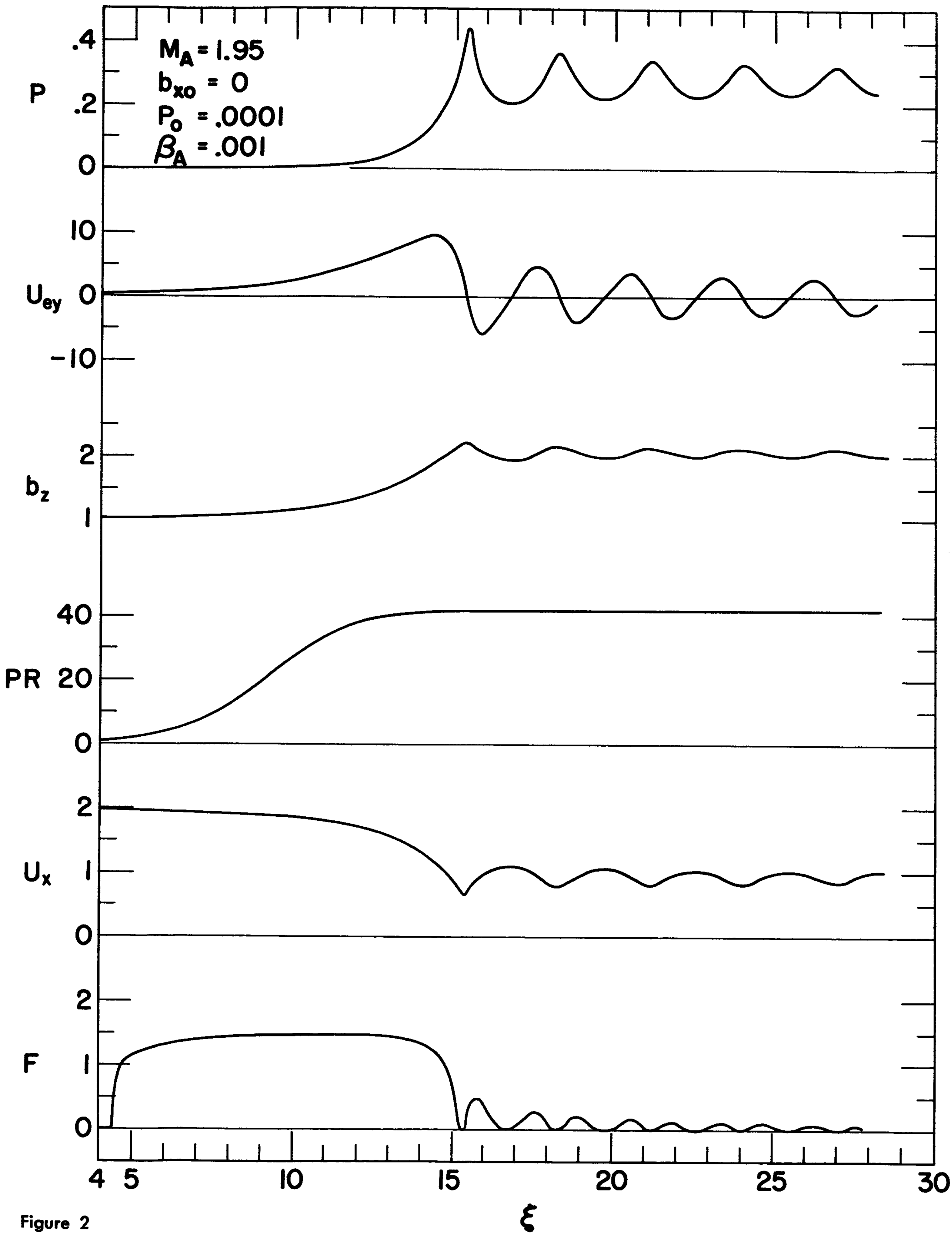

GROWTH RATE , R = 1836
.05
.025
0
$\frac{\omega_i}{\omega_{pe}}$
.25
.50
.75
1.0
1.25
kv/ω_{pe}

Figure 1



$M_A = 1.95$
$b_{xo} = 0$
$P_o = .0001$
$\beta_A = .001$
P
.4
.2
0
10
U_{ey}
0
−10
2
b_z
1
40
PR
20
0
2
U_x
1
0
2
F
1
0
4
5
10
15
20
25
30
ξ

Figure 2

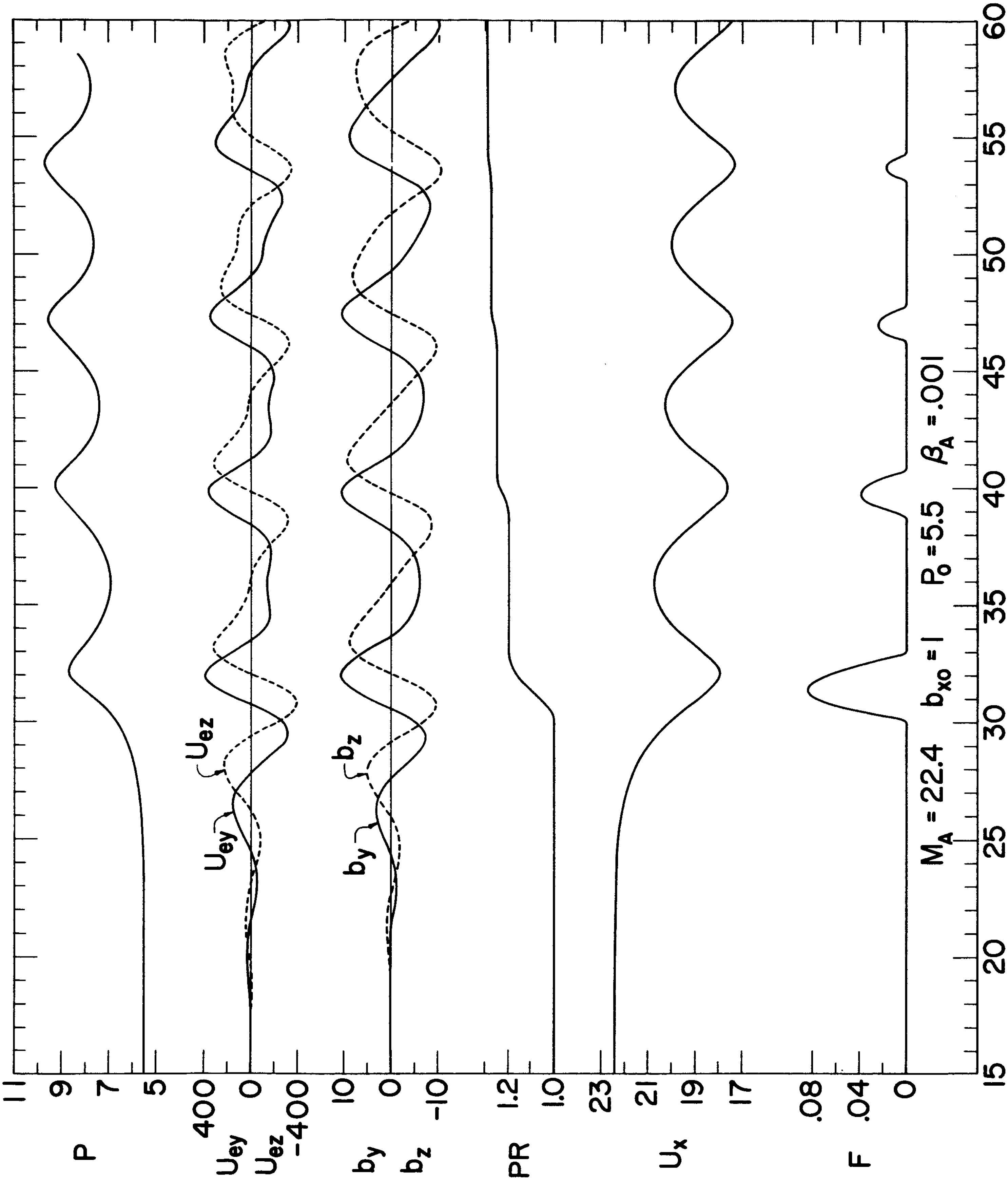


Figure 3

FIGURE CAPTIONS

Figure 1. The growth rate of the counter-streaming instability for zero temperature.

Figure 2. A shock at 1.95 times Alfven speed, with very low initial pressure, travelling perpendicular to the magnetic field.

Figure 3. A shock at $M_A = 22.4$, travelling along the magnetic field.

REFERENCES

1. Axford, W. N., JGR, 67, 3791, (1962).
2. Kellogg, P. J., JGR, 67, 3805, (1962).
3. Parker, E. N., Ap. J., 133, 1014, (1961).
4. Adlam, J. H. and J. E. Allen, Phil. Mag. 3, 448, (1958).
5. Buneman, O., Phys. Rev. Letters I, 8, (1958).
6. Stringer, T. E. Electrostatic Instabilities in Current Carrying and Counter Charging Plasmas, AERE Harwell, England (unpublished)
7. Kellogg, P. J., (unpublished)
8. Freeman, J. W., J. A. Van Allen and Cahill, L. J., JGR, 68, 2121, (1963).
9. Hain, K., Lüst, R. and Schlüter, A., Rev. Mod. Phys., 32, 967, (1960).
10. Sonett, C. P., Phys. Rev. Letters, 5, 46, (1960).
11. Cahill, L. J. and Amazeen, P. G., JGR, 68, 1835-1843, (1963).